

**28/April,
Wuhan**

Dark energy in scalar-tensor versus vector-tensor theories

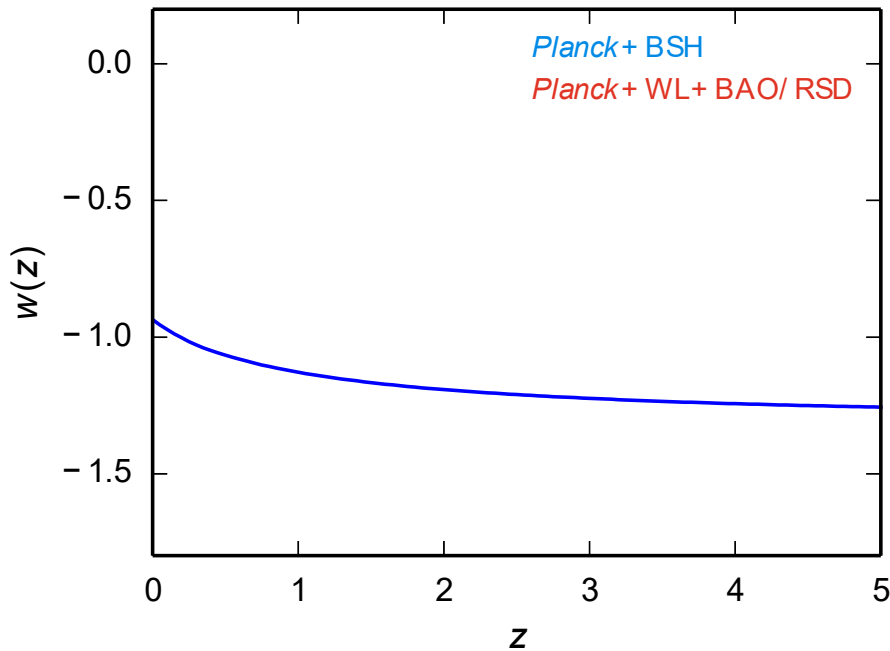
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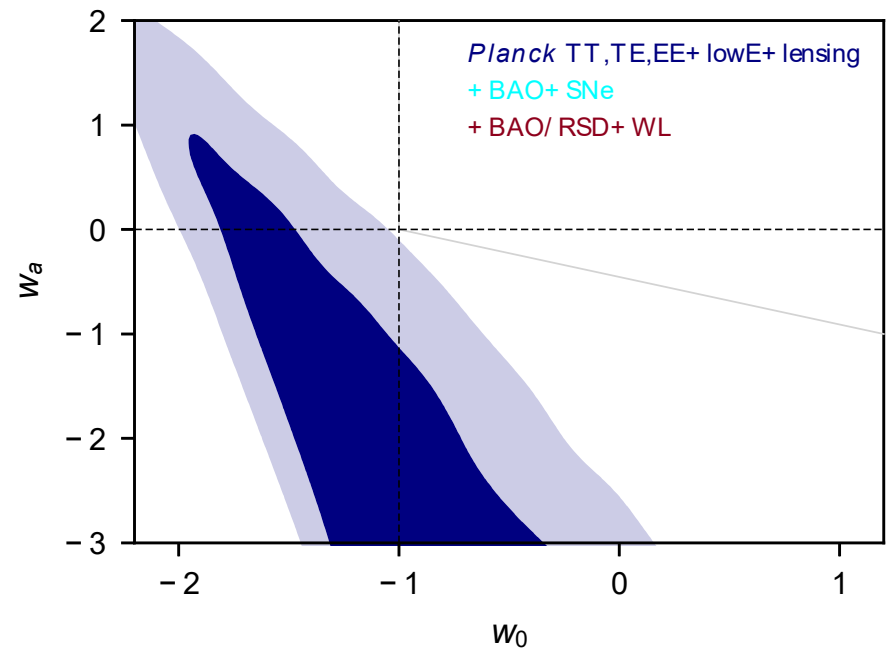
Observational constraints on the dark energy equation of state

For the time-varying equation of state $w(a) = w_0 + (1 - a)w_a$,

Planck2015+BAO+SNe+H0



Planck2018+BAO+SNe



The deviation from $w = -1$ (especially the region $w < -1$) is allowed from the data.

Theoretical models of cosmic acceleration

- Cosmo-illogical constant

$$w = -1$$

(by Rocky Kolb)

The vacuum energy can work as a cosmological constant, but it is difficult to explain the tiny observed dark energy scale.

- Dynamical dark energy models

w evolves in time.

Additional ingredients to those appearing in standard model of particle physics and General Relativity are taken into account. They can be

Scalar field, vector field, massive gravitons,...



They may be directly coupled or uncoupled to gravity.



Scalar-tensor theories

For a scalar field coupled to gravity, Horndeski theories are known as the most general theories with *second-order* equations of motion.

➔ Avoiding instabilities of Hamiltonians unbounded from below.

Horndeski Lagrangian

$$L = G_2(\phi, X) + G_3(\phi, X)\square\phi + G_4(\phi, X)R - 2G_{4,X}(\phi, X) [(\square\phi)^2 - \phi^{;\mu\nu}\phi_{;\mu\nu}] \\ + G_5(\phi, X)G_{\mu\nu}\phi^{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)[(\square\phi)^3 - 3(\square\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu\nu}\phi^{;\mu\sigma}\phi^{;\nu}_{;\sigma}]$$

Scalar field ϕ with a kinetic energy $X = -\partial^\mu\phi\partial_\mu\phi/2$.

G_2, G_3, G_4, G_5 are arbitrary functions of ϕ and X .

R : Ricci scalar, $G_{\mu\nu}$: Einstein tensor

Horndeski derived this Lagrangian at the age of 25 when he was the student of David Lovelock and it was rediscovered in 2011.

Second-order scalar-tensor field equations in a four-dimensional space

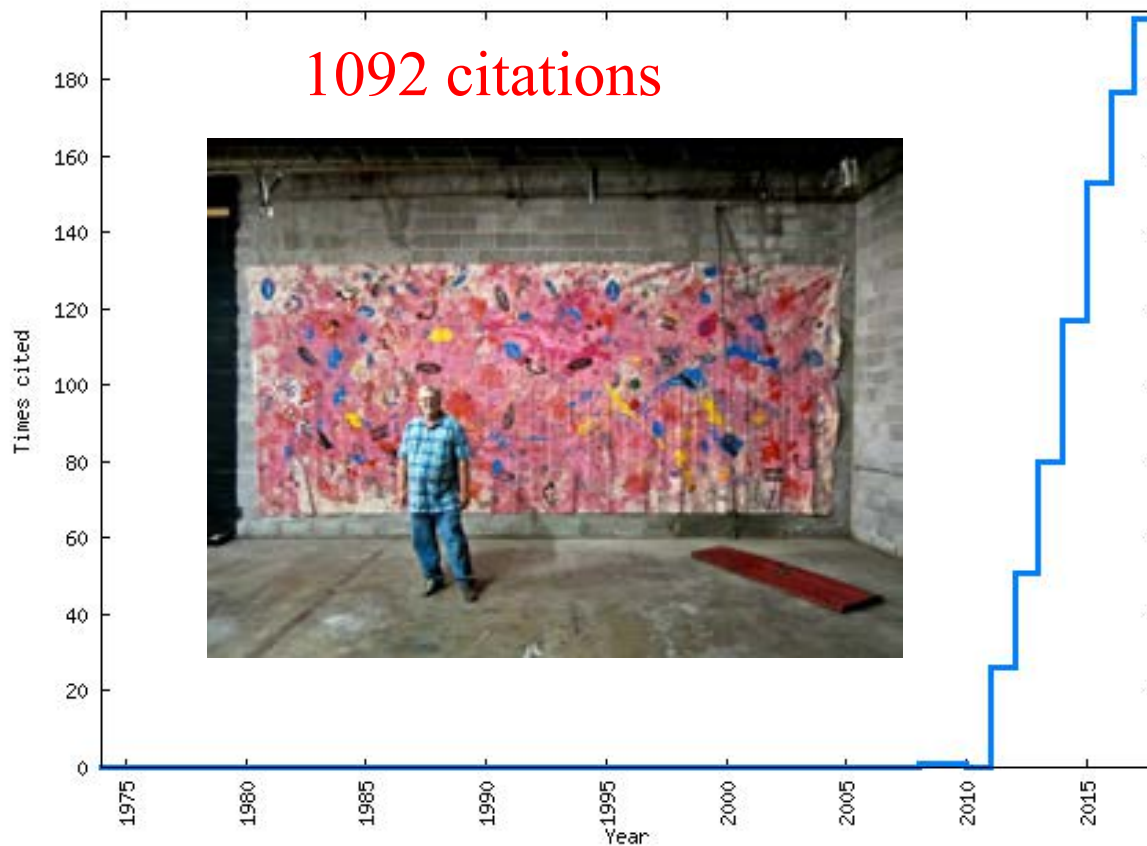
Gregory Walter Horndeski (Waterloo U.)

1974 - 21 pages

Int.J.Theor.Phys. 10 (1974) 363-384

DOI: [10.1007/BF01807638](https://doi.org/10.1007/BF01807638)

In 1983, Horndeski (1948~) quit physics and became an artist. He started to write papers on physics over the last four years (2015~).



Gravitational wave speed constraints on dark energy

The GW170817 event constrained the speed of gravitational waves to be very close to that of light.

$$-3 \times 10^{-15} \leq c_t/c - 1 \leq 7 \times 10^{-16}$$

Constraints on Horndeski theories

$$L = G_2(\phi, X) + G_3(\phi, X)\square\phi + G_4(\phi, X)R - 2G_{4,X}(\phi, X)[(\square\phi)^2 - \phi^{;\mu\nu}\phi_{;\mu\nu}] \\ + G_5(\phi, X)G_{\mu\nu}\phi^{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)[(\square\phi)^3 - 3(\square\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu\nu}\phi^{;\mu\sigma}\phi^{;\nu}_{;\sigma}]$$



Demanding that $c_t = c$

$$L = \underline{G_2(\phi, X)} + \underline{G_3(\phi, X)\square\phi} + \underline{G_4(\phi)R}$$

Quintessence,
K-essence

Cubic
Galileons

Brans-Dicke theory,
f(R) gravity

$$G_4 = M_{\text{pl}}^2/2 \text{ in GR}$$

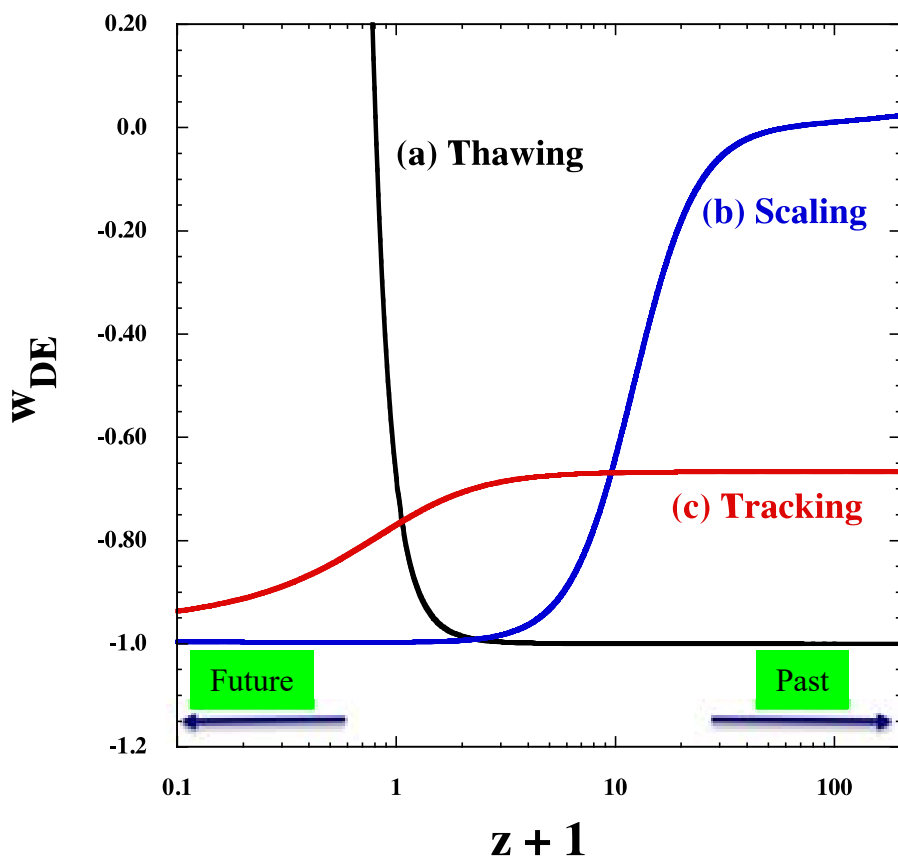
Current status of dark energy in scalar-tensor theories

● Quintessence and k-essence

➡ Minimally coupled to gravity

See Kase and ST (1809.08735) for review.

$$w > -1$$



(a) Thawing models are consistent with the data for $w < -0.7$ today.

(b) Scaling models are consistent with the data if the transition to the region $w = -1$ occurs for $z > 8$.

(c) Tracker models are consistent with the data for $w < -0.92$ during the matter era.

➡ For $V(\phi) \propto \phi^{-p}$, $p < 0.17$.

Brans-Dicke (BD) theory (1961)

Lagrangian: $L = \frac{M_{\text{pl}}^2}{2} F(\phi) R + (1 - 6Q^2) F(\phi) X$

where $F(\phi) = e^{-2Q\phi/M_{\text{pl}}}$

The constant Q characterizes the coupling between the scalar field ϕ and matter in the Einstein frame. It is related to the BD parameter ω_{BD} , as

$$Q^2 = \frac{1}{2(3 + 2\omega_{\text{BD}})} \quad \longrightarrow \quad \text{GR is recovered for } \omega_{\text{BD}} \rightarrow \infty \text{ i.e., } Q \rightarrow 0$$

The coupling Q mediates fifth forces. The solar system experiment gives

$$\omega_{\text{BD}} > 40000 \quad \longrightarrow \quad |Q| < 2.4 \times 10^{-3}$$

For $|Q| > 2.4 \times 10^{-3}$, we need some screening mechanism of fifth forces. Two examples are

(1) Chameleon mechanism: Based on the scalar potential $V(\phi)$

e.g., $f(R)$ gravity

(2) Vainshtein mechanism: Based on the derivative coupling $G_3(X) \square \phi$

e.g., Cubic Galileon $X \square \phi$



$f(R)$ gravity (chameleon mechanism)

Starobinsky (1980),
Capozziello (2003),
Carroll et al. (2003),
Nojiri and Odintsov (2004),
...

The $f(R)$ gravity is equivalent to BD theories with $Q = -1/\sqrt{6}$ in the presence of a scalar potential:

$$V = \frac{M_{\text{pl}}^2}{2} \left(R \frac{\partial f}{\partial R} - f \right)$$

with the scalar degree of freedom (scalon): $\phi = \sqrt{\frac{3}{2}} M_{\text{pl}} \ln \frac{\partial f}{\partial R}$

As long as the form of $f(R)$ is designed to have a large mass in regions of high density, the chameleon mechanism is at work.

Example:

$$f(R) = R - \lambda R_0 \frac{(R/R_0)^{2n}}{(R/R_0)^{2n} + 1}$$

(Hu and Sawicki, 2007)

In the high-density region ($R \gg R_0$), the scalaron mass squared grows as

$$M_\phi^2 = \frac{d^2 V}{d\phi^2} \propto R^{2(n+1)} \gg H^2$$



The field is very heavy, so the propagation of fifth forces is suppressed.

f(R) dark energy

More than 1000 papers,
see De Felice and ST (2010).

The models are constructed to recover the Λ CDM behavior in the past.

$$f(R) = R - \lambda R_0 \frac{(R/R_0)^{2n}}{(R/R_0)^{2n} + 1} \quad \longrightarrow \quad f(R) = R - \lambda R_0 \quad \text{for } R \gg R_0$$

(Hu and Sawicki, 2007)

After R decreases to the order of R_0 , the model deviates from the Λ CDM.

Deviation parameter from the Λ CDM:

$$B = \frac{R f_{,RR}}{f_{,R}} \frac{H \dot{R}}{\dot{H} R} \quad \longrightarrow \quad B < 1.1 \times 10^{-3} \quad \text{today}$$

Lombriser et al (2012)

To avoid the large enhancement of perturbations at the late cosmological epoch ($G_{\text{eff}} = 4G/3$).

The variation of w at low redshifts is also limited:

$$|w + 1| < \mathcal{O}(0.01) \quad \longrightarrow \quad \text{Indistinguishable from the } \Lambda\text{CDM in current observations.}$$

Battye et al (2018)

Galileons and their extensions

If there are no signatures of nonminimal couplings, the left-over Horndeski Lagrangian is

$$L = G_2(\phi, X) + G_3(\phi, X)\square\phi + \frac{M_{\text{pl}}^2}{2}R$$

There are three possibilities (in the presence of cubic Lagrangian):

(A) Galileons without a potential:

$$L = X + \frac{\beta_3}{M^3}X\square\phi + \frac{M_{\text{pl}}^2}{2}R$$

➡ Ruled out

There exists the self-accelerating solution with $\dot{\phi} = \text{constant}$.

(B) Galileons with a potential:

$$L = X - V(\phi) + \frac{\beta_3}{M^3}X\square\phi + \frac{M_{\text{pl}}^2}{2}R$$

Galileon has a linear potential $V(\phi) = m^3\phi$ driving cosmic acceleration.

(C) Galileons with k-essence:

$$L = G_2(X) + \frac{\beta_3}{M^3}X\square\phi + \frac{M_{\text{pl}}^2}{2}R$$

For example, the ghost condensate $G_2(X) = -X + c_2X^2$ leads to the dark energy dynamics different from case (A).

(A) Galileons without a potential

Lagrangian: $L = X + \frac{\beta_3}{M^3} X \square \phi + \frac{M_{\text{pl}}^2}{2} R$

There is a tracker solution along which $w = -2$ in the matter era (finally approaching $w = -1$).

De Felice and ST (2010)



Disfavored from the CMB+BAO+SNe data

Nesseris, De Felice, ST (2010)

The Galileon gives rise to the cosmic growth rate larger than that in GR.

Newtonian gravitational potential Ψ : $\frac{k^2}{a^2} \Psi = -4\pi G \mu \delta \rho_m$

Weak lensing gravitational potential ψ_{eff} : $\frac{k^2}{a^2} \psi_{\text{eff}} = 8\pi G \Sigma \delta \rho_m$

$\mu = \Sigma > 1$ for Galileons

The cosmic growth history of Galileons is in tension with the observational data of redshift-space distortions, weak lensing, and ISW-galaxy cross-correlations.

Renk et al (2016)

Extended Galileons and ISW-galaxy cross-correlations

$$L = X + \frac{\beta_3}{M^{4n-1}} X^n \square \phi + \frac{M_{\text{pl}}^2}{2} R$$

De Felice and ST (2011)

Correlations between the effect in CMB and galaxy distributions

For $n = \mathcal{O}(1)$, Σ rapidly grows in time.



Correlations between $\dot{\psi}_{\text{eff}}$ and $\delta\rho_m$

The models with small n like $n = 1$ leads to the negative cross-correlation incompatible with the data.

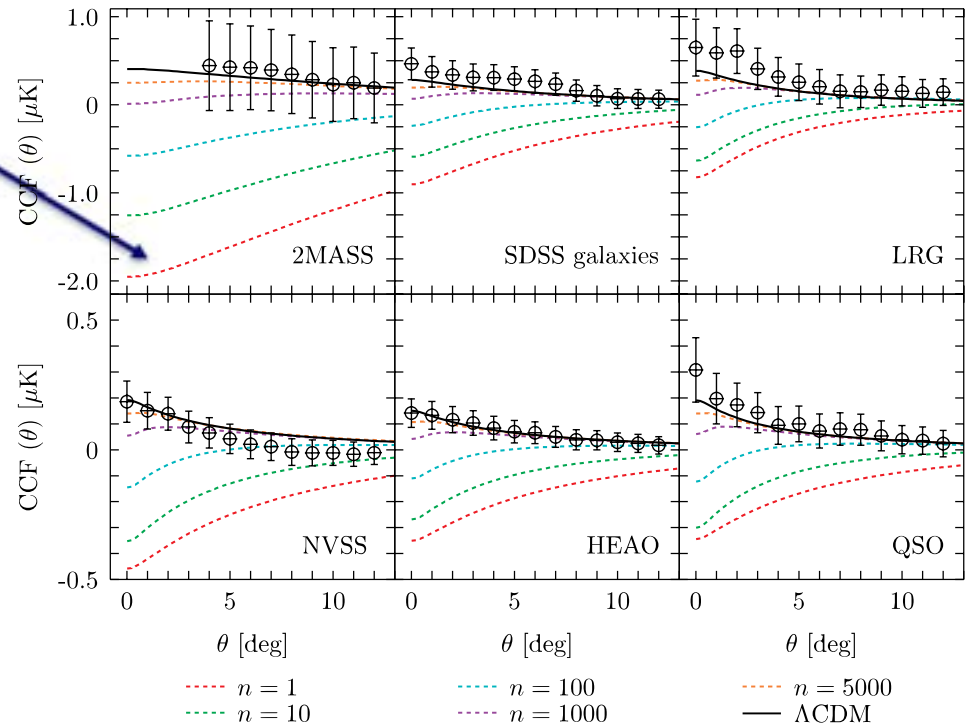
The ISW-galaxy data constrain the power in the range

$$n \gtrsim \mathcal{O}(100)$$



Excluding cubic Galileons ($n=1$)

Cross-correlation amplitude versus angle



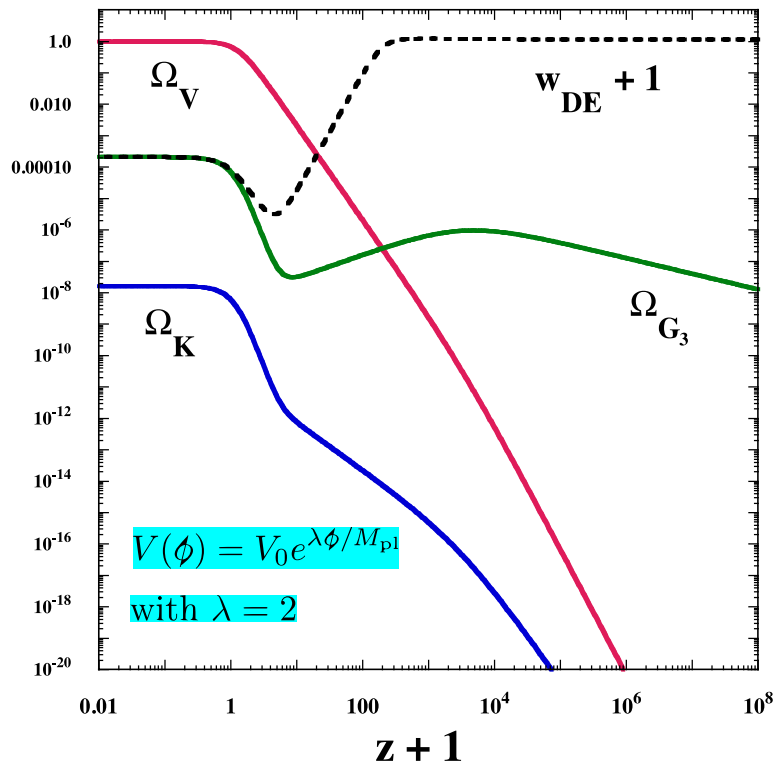
R. Kimura, T. Kobayashi and K. Yamamoto (2012)

(B) Galileons with a potential

$$L = X - V(\phi) + \frac{\beta_3}{M^3} X \square \phi + \frac{M_{\text{pl}}^2}{2} R$$

Provided the potential $V(\phi)$ of a light scalar dominates over the Galileon term at late times, the model is observationally allowed.

Bound on today's Galileon density parameter: $\Omega_{G_3}(t_0) < 0.2$



For $\beta_3 > 1$, the Galileon term can suppress the field kinetic energy such that $\Omega_K \ll \Omega_{G_3} \ll \Omega_V = \mathcal{O}(1)$ today.

Even for $\lambda \equiv M_{\text{pl}} V_{,\phi} / V > 1$, the dark energy equation of state quickly approaches -1 after the dominance of Ω_V (with $w_{\text{DE}} > -1$).

This model predicts

$$w_{\text{DE}} > -1$$

(C) Galileons with k-essence

Kase and ST (2018)

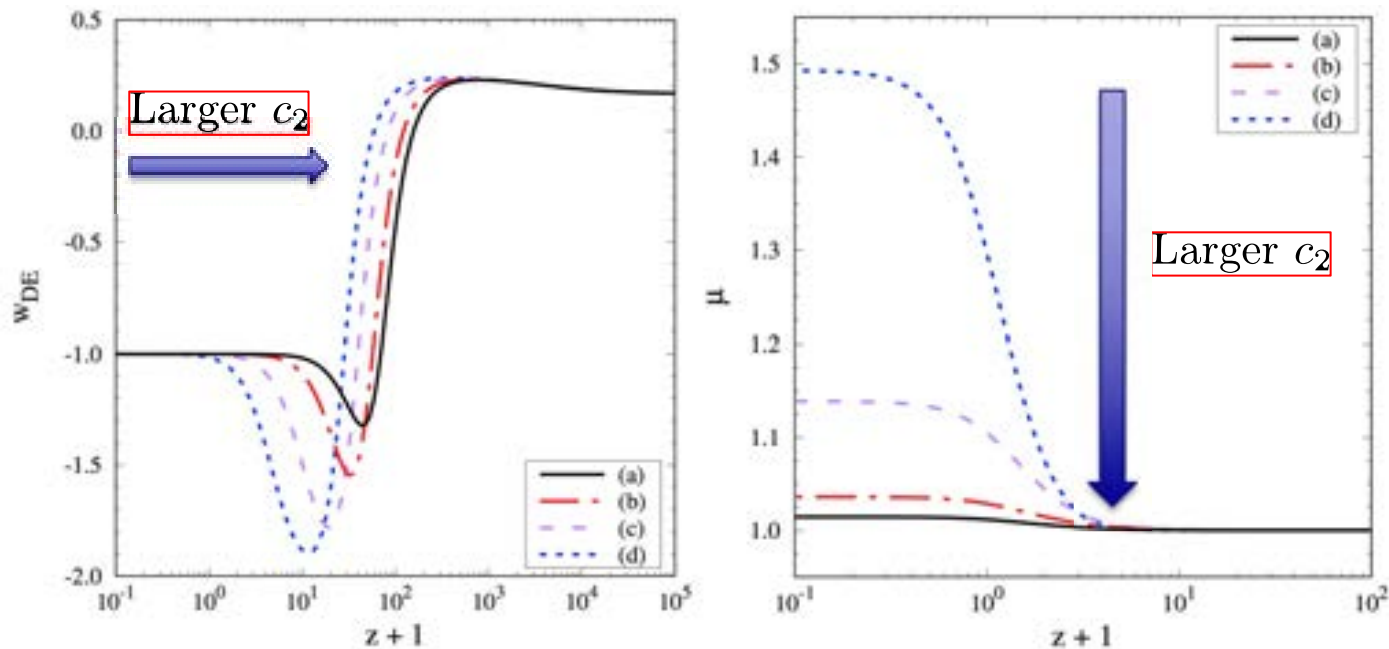
$$L = X + c_2 X^2 + \frac{\beta_3}{M^3} \square \phi + \frac{M_{\text{pl}}^2}{2} R$$

Ghost condensate + Galileon



This term prevents the approach to tracker solutions ($w_{\text{DE}} = -2$).

Moreover, the growth perturbations can be close to that in GR ($\mu = \Sigma = 1$).



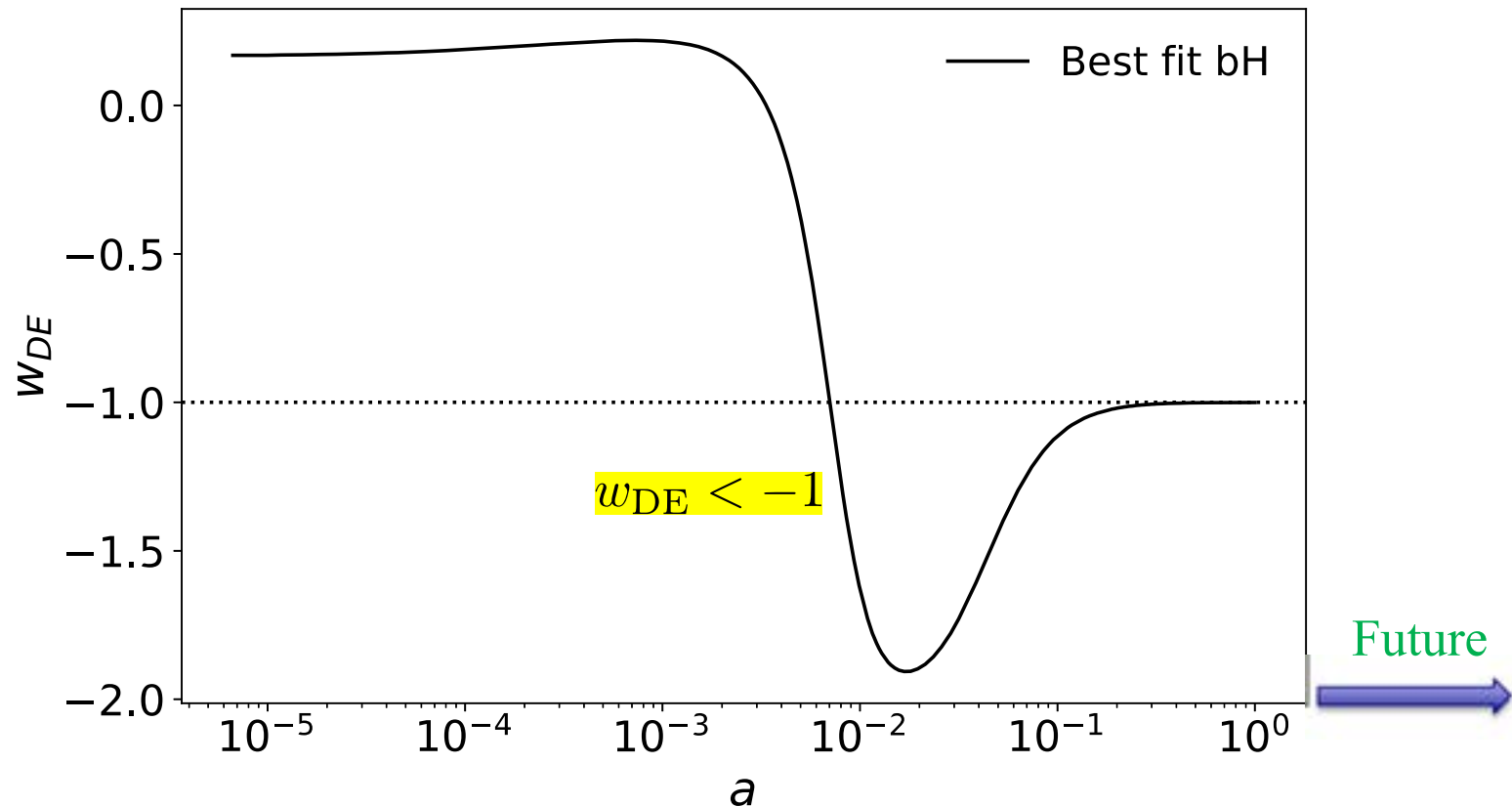
This model enters the region $w_{\text{DE}} < -1$

Observational constraints on Model (C)

Peirone, Benevento, Frusicante, ST, in preparation

(CMB+BAO+SN Ia+RSD)

The model entering the region $w_{DE} < -1$ exhibits the better fit to the data relative to the Λ CDM model.



Short summary of scalar-tensor dark energy

The GW170817 event constrained the Horndeski Lagrangian to be

$$L = G_2(\phi, X) + G_3(\phi, X)\square\phi + G_4(\phi)R$$

- So far, there were no observational signatures for nonminimally coupled theories (including $f(R)$ gravity).
- The cubic Galileon with the late-time dominance as dark energy is ruled out from observations.
- The cubic Galileon with a potential or with k-essence should leave observational signatures consistent with current observations.



If these models are ruled out from future observations, the allowed dark energy theories reduce to quintessence, k-essence, or the cosmological constant.



Part II

Dark energy in vector-tensor theories

Vector fields can derive cosmic acceleration?

Massless and massive vector fields A_μ in Minkowski space-time

(i) Maxwell field (massless)

Lagrangian: $\mathcal{L}_F = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

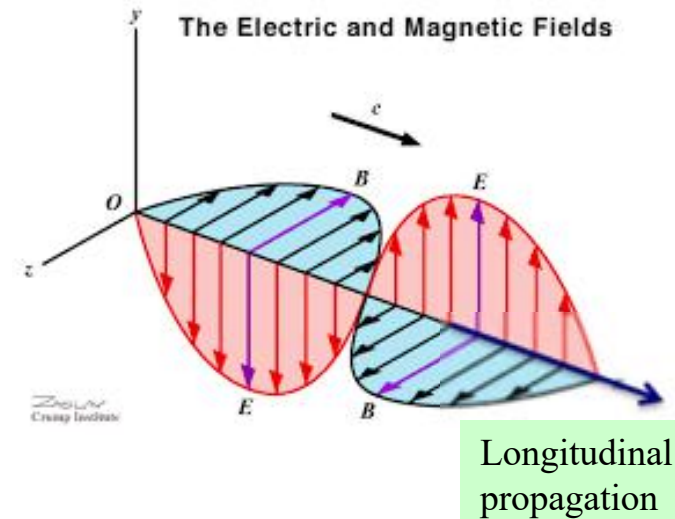
There are two transverse polarizations (electric and magnetic fields).

(ii) Proca field (massive)

Lagrangian: $\mathcal{L}_F = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu$

Introduction of the mass m of the vector field A_μ allows the propagation in the longitudinal direction due to the breaking of $U(1)$ gauge invariance.

2 transverse and 1 longitudinal
= 3 DOFs



Vector-tensor theories



On general curved backgrounds, it is possible to extend the massive Proca theories to those containing three DOFs and two tensor polarizations.

Heisenberg (2014), Tasinato (2014)

$$\begin{aligned}
 \mathcal{L}_2 &= G_2(X, F, Y), & \longrightarrow & \text{Intrinsic vector mode in } F \text{ and } Y \\
 \mathcal{L}_3 &= G_3(X) \nabla_\mu A^\mu, \\
 \mathcal{L}_4 &= G_4(X) R + G_{4,X}(X) [(\nabla_\mu A^\mu)^2 - \nabla_\rho A_\sigma \nabla^\sigma A^\rho], \\
 \mathcal{L}_5 &= G_5(X) G_{\mu\nu} \nabla^\mu A^\nu - \frac{1}{6} G_{5,X}(X) [(\nabla_\mu A^\mu)^3 - 3 \nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\sigma A^\rho + 2 \nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla^\sigma A_\gamma] \\
 &\quad - g_5(X) \tilde{F}^{\alpha\mu} \tilde{F}^{\beta}{}_\mu \nabla_\alpha A_\beta, \\
 \mathcal{L}_6 &= G_6(X) L^{\mu\nu\alpha\beta} \nabla_\mu A_\nu \nabla_\alpha A_\beta + \frac{1}{2} G_{6,X}(X) \tilde{F}^{\alpha\beta} \tilde{F}^{\mu\nu} \nabla_\alpha A_\mu \nabla_\beta A_\nu, & \left. \vphantom{\mathcal{L}_5} \right\} & \text{Intrinsic vector mode}
 \end{aligned}$$

where $X = -\frac{1}{2} A_\mu A^\mu$, $F = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$, $Y = A^\mu A^\nu F_\mu{}^\alpha F_{\nu\alpha}$

$$L^{\mu\nu\alpha\beta} = \frac{1}{4} \epsilon^{\mu\nu\rho\sigma} \epsilon^{\alpha\beta\gamma\delta} R_{\rho\sigma\gamma\delta}, \quad \tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$$

Taking the scalar limit $A^\mu \rightarrow \nabla^\mu \pi$, the above Lagrangian recovers a sub-class of Horndeski theories (with \mathcal{L}_6 vanishing).

In 1976, Horndeski derived the U(1)-invariant interaction: $G_6(X) = \text{constant}$.

U(1) gauge-invariant interaction: constant G_6

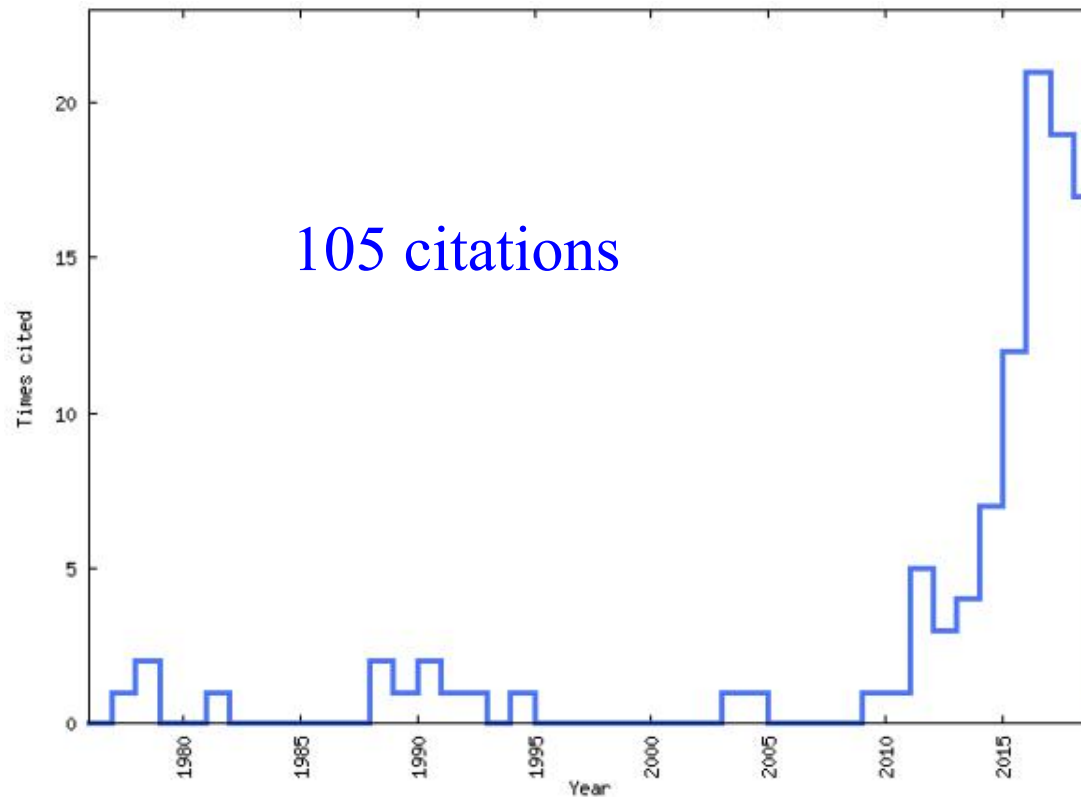
Conservation of Charge and the Einstein-Maxwell Field Equations

G.W. Horndeski (Waterloo U.)

1976 - 8 pages

J.Math.Phys. 17 (1976) 1980-1987

DOI: [10.1063/1.522837](https://doi.org/10.1063/1.522837)



After the GW170187 event : $C_t = C$

$$\mathcal{L}_2 = G_2(X, F, Y),$$

$$\mathcal{L}_3 = G_3(X) \nabla_\mu A^\mu,$$

$$\mathcal{L}_4 = G_4(X) R + G_{4,X}(X) [(\nabla_\mu A^\mu)^2 - \nabla_\rho A_\sigma \nabla^\sigma A^\rho],$$

$$\mathcal{L}_5 = G_5(X) G_{\mu\nu} \nabla^\mu A^\nu - \frac{1}{6} G_{5,X}(X) [(\nabla_\mu A^\mu)^3 - 3 \nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\sigma A^\rho + 2 \nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla^\sigma A_\gamma] - g_5(X) \tilde{F}^{\alpha\mu} \tilde{F}^{\beta}_{\mu} \nabla_\alpha A_\beta,$$

$$\mathcal{L}_6 = G_6(X) L^{\mu\nu\alpha\beta} \nabla_\mu A_\nu \nabla_\alpha A_\beta + \frac{1}{2} G_{6,X}(X) \tilde{F}^{\alpha\beta} \tilde{F}^{\mu\nu} \nabla_\alpha A_\mu \nabla_\beta A_\nu,$$

} Intrinsic
vector modes

- $G_4(X)$ needs to be constant.
- Intrinsic vector modes (including F, Y dependence in G_2) survive.

A simple dark energy model in vector-tensor theories

$$S = \int d^4x \sqrt{-g} \left[F + G_2(X) + G_3(X) \nabla_\mu A^\mu + \frac{M_{\text{Pl}}^2}{2} R \right] + S_M$$

$$F = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu},$$

$$X = -\frac{1}{2} A_\mu A^\mu$$

where

$$G_2(X) = b_2 X^{p_2}, \quad G_3(X) = b_3 X^{p_3}$$

On the FLRW background, the temporal and spatial components of A^μ are

$$A^0 = \underbrace{\phi(t)}_{\text{Background value}} + \delta\phi, \quad A^i = \frac{1}{a^2} \delta^{ij} (\partial_j \chi_V + E_j)$$

↙ ↘

The background temporal component (auxiliary field) obeys

$$\phi^p H = \text{const.} \quad \text{where} \quad p = 1 - 2p_2 + 2p_3$$

➔ For $p > 0$, ϕ grows with the decrease of H and it approaches the de Sitter attractor (constant H).

Dark energy equation of state (background evolution)

$$w_{\text{DE}} = -\frac{3(1+s) + s\Omega_r}{3(1+s\Omega_{\text{DE}})}$$



- (a) $w_{\text{DE}} = -1 - 4s/3$ in the radiation era,
- (b) $w_{\text{DE}} = -1 - s$ in the matter era,
- (c) $w_{\text{DE}} = -1$ in the de Sitter era

$s = p_2/p$ characterizes the deviation from $w_{\text{DE}} = -1$.

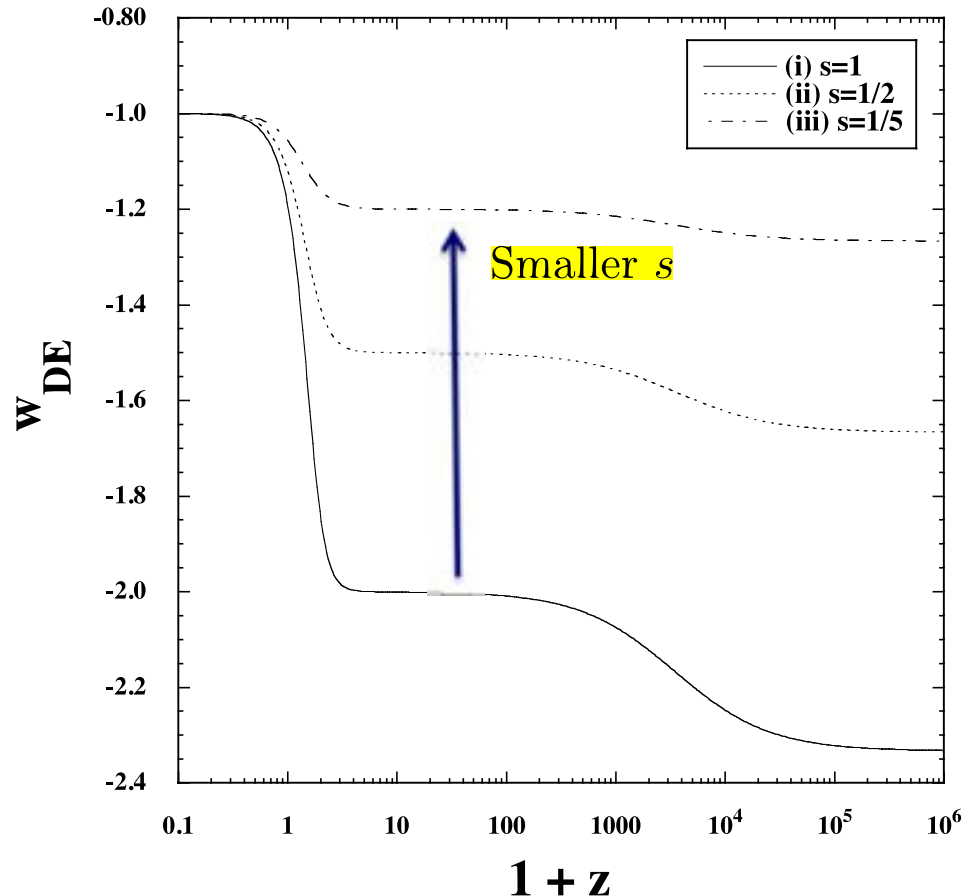
The joint analysis based on SNIa, CMB, BAO, H_0 data place the bound

$$s = 0.254^{+0.118}_{-0.097} \quad (95\% \text{ CL})$$

De Felice, Heisenberg, ST (2017)

$s = 0$ is outside the 95% CL border.

The phantom behavior of w_{DE} reduces the tension of H_0 between high and low redshift measurements that exists for the Λ CDM ($s = 0$).



Cosmological perturbations

The spatial vector component A^i contains scalar and vector perturbations.

$$A^i = \frac{1}{a^2} \delta^{ij} (\partial_j \chi_V + E_j)$$

Longitudinal scalar perturbations

Intrinsic vector perturbations

Both χ_V and E_j affect the evolution of gravitational potentials Ψ, Φ and the matter density perturbation $\delta\rho_m$ through

Newtonian potential:

$$\frac{k^2}{a^2} \Psi = -4\pi G \mu \delta\rho_m$$

Weak lensing potential $\psi_{\text{eff}} = \Phi - \Psi$:

$$\frac{k^2}{a^2} \psi_{\text{eff}} = 8\pi G \Sigma \delta\rho_m$$

The matter density contrast δ_m obeys

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi\mu G\rho_m \delta_m \simeq 0$$

The deviations of μ and Σ from 1 lead to the modified evolution of $\Psi, \psi_{\text{eff}}, \delta_m$ compared to GR.

Cosmic growth rate in vector-tensor theories

On the de Sitter attractor, the two dimensionless gravitational couplings reduce to

$$\mu_{\text{dS}} = \Sigma_{\text{dS}} = 1 + \left[\frac{1 - ps}{ps} + \left(\frac{2}{3^{1/p}} \right)^{1/(1+s)} \frac{1}{\lambda_V} \right]^{-1}$$

where λ_V is associated with the intrinsic vector mode such that

$$\lambda_V = \left[\left(\frac{\phi}{M_{\text{pl}}} \right)^p \frac{H}{m} \right]^{2/[p(1+s)]}$$

q_V $q_V = 1$ in our model
(coefficient of kinetic term
of vector perturbations)

↳ In the limit $\lambda_V \rightarrow 0$, $\mu_{\text{dS}} = \Sigma_{\text{dS}} \rightarrow 1$
The evolution of perturbation is similar to that in GR.

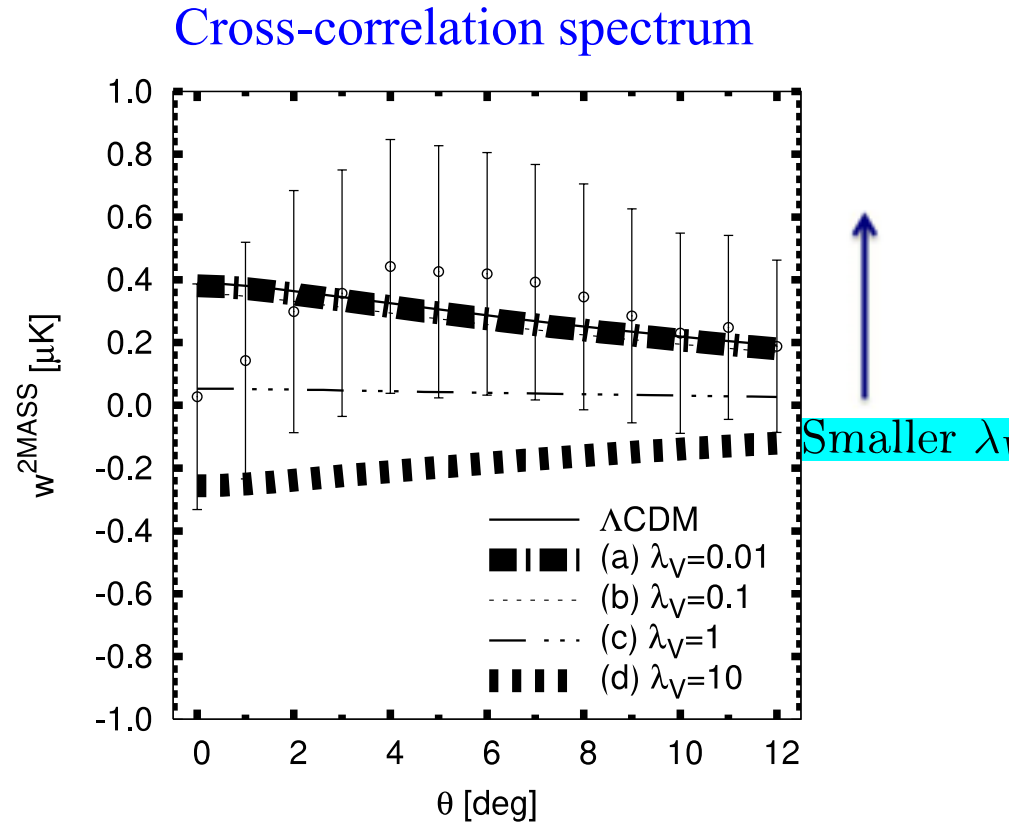
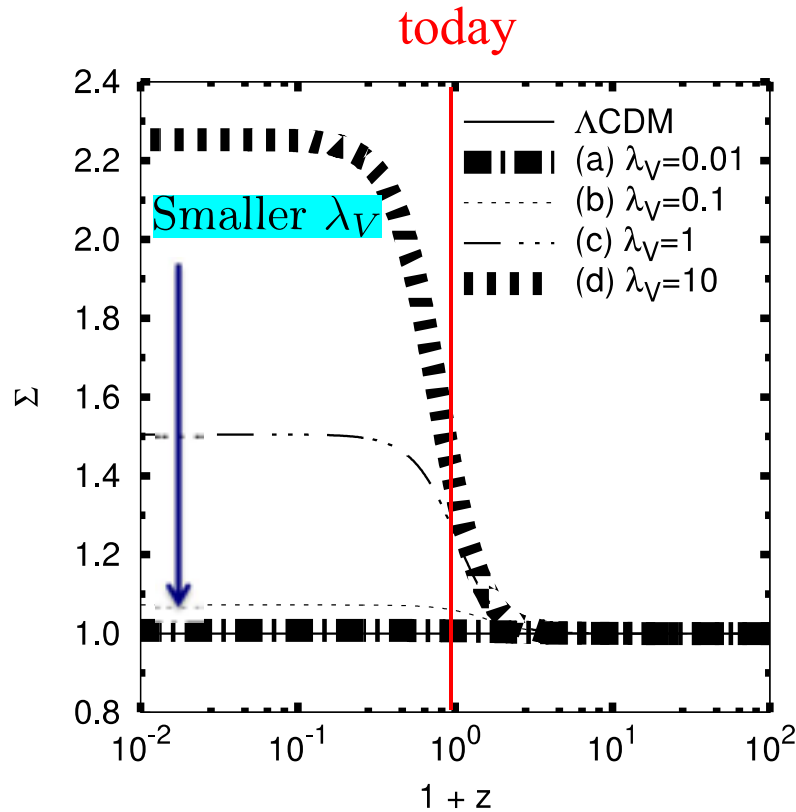
➡ Compatible with growth-rate measurements

↳ In the limit $\lambda_V \rightarrow \infty$, $\mu_{\text{dS}} = \Sigma_{\text{dS}} > 1$
This case reduces to a subclass of scalar-tensor theories.

➡ Difficult to be compatible with growth-rate measurements

ISW-galaxy cross-correlations

For smaller λ_V , the model shows a better compatibility with the data.

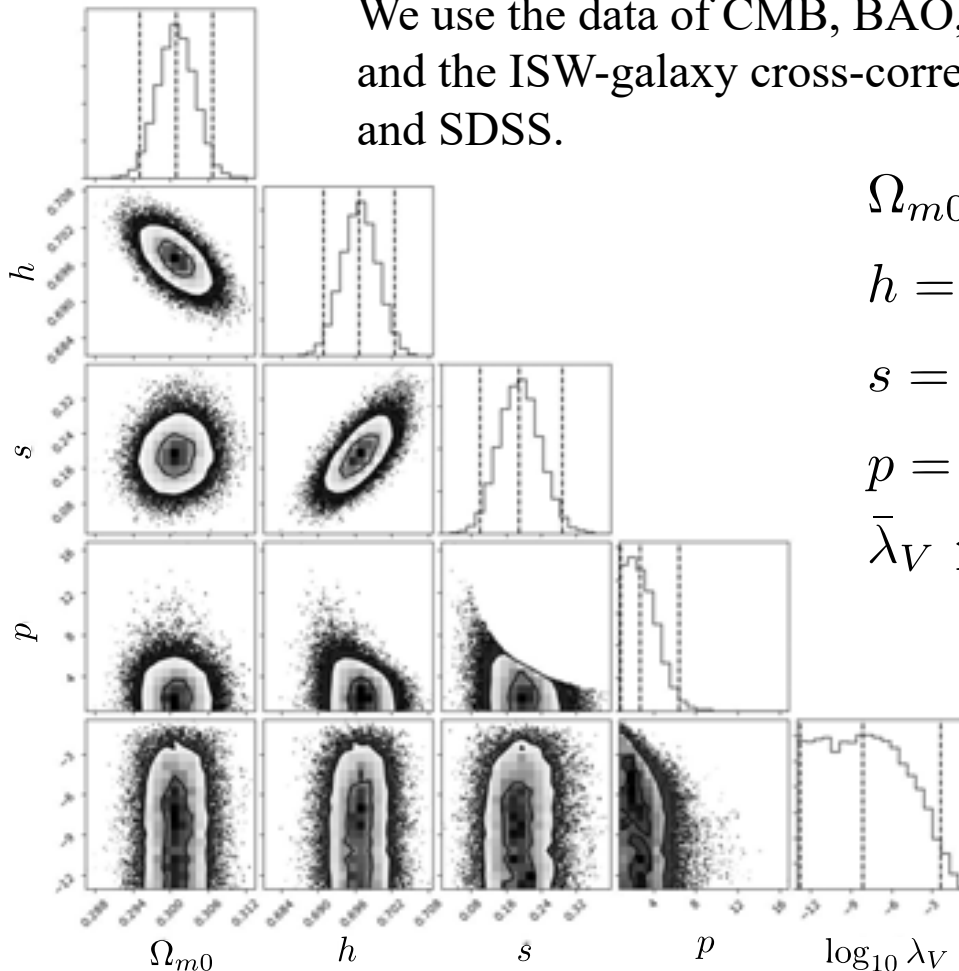


The existence of intrinsic vector mode in generalized Proca theories can give rise to **positive** cross-correlations compatible with the data.

Observational constraints on generalized Proca theories

Nakamura, De Felice, Kase, ST (2018)

We use the data of CMB, BAO, SN Ia, H0, redshift space distortions, and the ISW-galaxy cross-correlations with the catalogues of 2MASS and SDSS.



$$\Omega_{m0} = 0.301^{+0.006}_{-0.006},$$

$$h = 0.697^{+0.006}_{-0.006},$$

$$s = 0.185^{+0.100}_{-0.089},$$

$$p = 3.078^{+4.317}_{-2.119},$$

$$\bar{\lambda}_V \leq \lambda_V < 0.015, \quad (95\% \text{ CL})$$

Deviation from Λ CDM model

$\phi^p \propto H^{-1}$

The ISW-galaxy data gives this upper bound.

The model with $s > 0$ still fits the data better than the Λ CDM model.

$$\text{Best-fit: } \chi^2_{\min} = 618.9$$

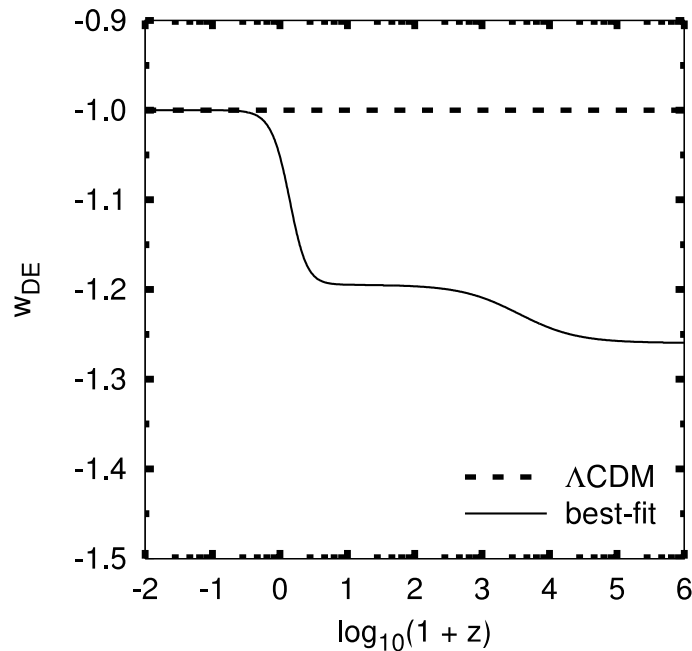
$$\Lambda\text{CDM: } \chi^2_{\min} = 642.7$$

Best-fit model

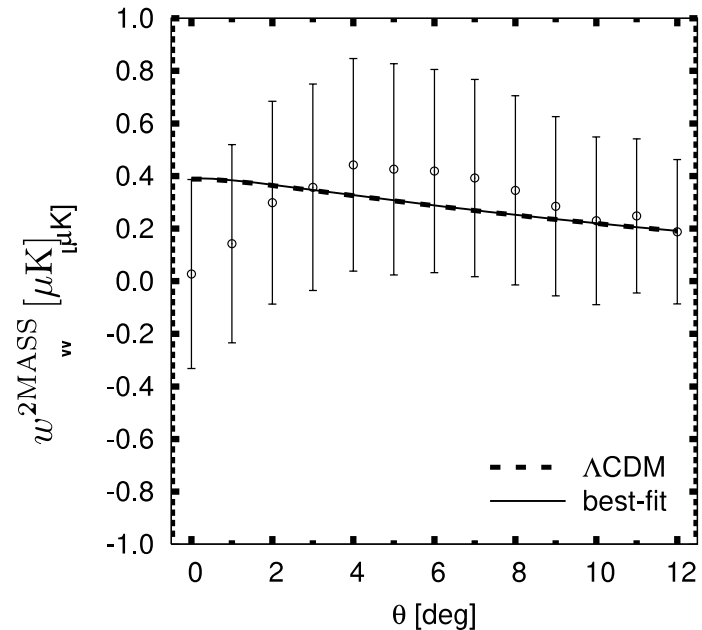
$$\Omega_{m0} = 0.301, h = 0.697, s = 0.185, p = 3.078, \log_{10} \lambda_V = -7.359 \quad \Rightarrow \quad \chi^2 = 618.9.$$

The background dynamics in our model is different from that in the Λ CDM model, while the perturbation dynamics is similar to that in Λ CDM.

$$w_{\text{DE}} = -\frac{3(1+s) + s\Omega_r}{3(1+s\Omega_{\text{DE}})}$$



$$w(\theta) \equiv T_{\text{CMB}} \sum_{l=0}^{\infty} \frac{2l+1}{4\pi} C_l^{\text{IG}} \mathcal{P}_l(\cos \theta)$$



Akaike-information-criterion (AIC): $\text{AIC} = \chi^2_{\text{min}} + 2 \times 5 = 628.9$

Smaller than $\text{AIC}_{\Lambda\text{CDM}} = \chi^2_{\text{min},\Lambda\text{CDM}} + 2 \times 2 = 646.7$

Summary

- The GW170817 event placed tight constraints on dark energy models in scalar-tensor and vector-tensor theories.
- Unless the potential or k-essence terms are taken into account, the *scalar* cubic Galileon is excluded from the data (especially from the negative ISW-galaxy cross-correlations).
- For the *vector* cubic Galileon, the ISW-galaxy cross-correlation can be positive due to the existence of intrinsic vector modes.
- In vector-tensor theories, the model with $s > 0$ fits the data better than the Λ CDM model by reducing the tension of H_0 between high- and low-redshift measurements.

Let's see whether or not future observations may find some signatures for the deviation from the Λ CDM model.